MWA2018 Course Guide

Moscow Winter Academy

Age-structured modeling and management of biological and economic resources

3-6 February 2018

Program Committee:

Elena Rovenskaya, International Institute for Applied Systems Analysis (Austria) & Lomonosov Moscow State University (Russia), Anne Maria Eikeset, University of Oslo (Norway), Nikolay Grigorenko, Lomonosov Moscow State University (Russia), Simon Levin, Princeton University (USA), Yuri Osipov, Lomonosov Moscow State University (Russia), Andries Peter Richter, University of Oslo (Norway) & Wageningen University (The Netherlands), Nils Christian Stenseth, University of Oslo (Norway)

MWA2018 Concept

The course is dedicated to modeling and management of biological and economic resources in cases, when accounting for heterogeneity within a population in terms of age or/and size is important to get the system's dynamics right.

In nature, both animal and plant individuals go through the stages of being born, growing up, maturing, giving life to newborns, ageing and eventual dying. The age of an individual is key to characterize its ability to grow and reproduce; age also defines the probability of an individual to die. Hence, models, in which these vital parameters are dependent on age, are more realistic to describe the population dynamics.

Besides age, growth, fecundity and mortality of an individual depend also on the amount of critical resources, which the individual consumes. In natural conditions, these critical resources come from the environment: Trees need water and minerals from soil, as well as sunlight for photosynthesis; fish need calories, which they can get by consuming phytoplankton, zooplankton, smaller fish or other water inhabitants; etc. The amount of resources, which an individual can get from the environment, depends on the individual's age and size: Bigger trees shade smaller fellow, thus receiving more sunlight, bigger fish can feed on a large food base than smaller fish. The models should take this dependence into account.

Moreover, what also matters here is the total population density. Since the amount of the available critical resources is naturally limited, the total number of individuals (=the population size) competing for the given resource base matters to define how much resources each individual will get.

This course considers mathematical models, simulating population dynamics, taking into account the age and size effects together with the population density dependence. It relies of the "transport" equation, which is a first order partial differential equation, whose coefficients are non-linear functions of time, individual's size/age, and the total population density. The boundary condition, defining the number of newborns, is based on the size of the matured population. These dependencies make the system "non-local", which in turn makes the mathematics behind such models rather non-trivial. MWA2018 will address the questions of solution definition, existence and uniqueness.

Furthermore, we will consider not only natural (~undisturbed) populations, but also systems, which are subject to exploitation. Depending on the ownership structure and governmental policies, a forest manager might be interested in maximizing the economic profits from timber production, or she might also consider other ecosystem services, like carbon sequestration, for example. In case of fisheries, decisions makers might care about revenues or sustainable yields. In any case, as decision maker problem can be formulated in an optimal control setting, in which one selects an objective function to be optimized (maximized or minimized) under a set of constraints determined by the system dynamics. Controls are typically cutting rates; also plating rate can be a control.

In MWA2018 we consider application-driven classes of optimal control problems for first-order ODEs and formulate necessary conditions for optimality in the form akin to the Pontryagin Maximum Principle (PMP). In some cases, these necessary conditions allow deriving an optimal solution analytically; in more complicated cases they can enable obtaining a structure of an optimal control analytically.

MWA2018 focuses extensively on the application of these mathematical constructs to modeling forest dynamics. However, the same or very similar mathematical models can also be used to model economic dynamics. In MWA2018 we consider economic models, in which the capital stock ages over time and is replenished through investments. Here, an optimal control problem is formulated to find out what is an optimal strategy of a central planner, who aims to optimize her discounted consumption-based utility over an infinite time interval. She is facing a tradeoff between higher consumption today vis-à-vis bigger investment, which will lead to a greater output in future, which in turn will enable higher consumption levels.

Lecturers

- **Dr. Liudmila Artemieva, Lomonosov Moscow State University, Russia**
- **Dr. Professor Dr. Anton Belyakov, Lomonosov Moscow State University, Russia**
- **Professor Dr. Alexey Davydov, Lomonosov Moscow State University, Russia & National University of Science and Technology MISiS, Russia & International Institute for Applied Systems Analysis, Austria**
- **Professor Dr. Natali Hritonenko, Prairier View A&M University, TX, USA**
- **Dr. Elena Rovenskaya, International Institute for Applied Systems Analysis, Austria & Lomonosov Moscow State University, Russia**
- **Professor Dr. Nikolay Strigul, Washington State University Vancouver, WA, USA**
- **Dr. Nadezhda Vasilieva, Dokuchaev Soil Science Institute, Russia**

- Professor Dr. Edwin van der Werf, **Wageningen University, The Netherlands**

Basic Preparatory Reading

Pierre Magal, Shigui Ruan (Eds.): Structured Population Models in Biology and Epidemiology. Lectures in Mathematics. Springer-Verlag Berlin Heidelberg, 2008

Educational Lectures

Age- and Size-structured Modeling and Control in Economics, Biological and Environmental Sciences

- **Natali Hritonenko**

Part I. Vintage capital model

The versatility and challenges of constructing mathematical models will be discussed. Elements of qualitative analysis will be demonstrated on a continuous age- structured model important to economic growth theory. Emphasis will be made on the different ways to modify the model and on the applied interpretation of the obtained qualitative results.

Part II. Vintage capital model – extensions

Three models important to different applications and relation between models will be considered.

Part III. Age- size- structured population models

Age- and size- dependent population models with and without harvesting control (rate or effort) will be constructed and compared. Elements of their qualitative analysis and interpretation of outcomes will be discussed. Environmental protection and effects of climate change and natural disturbances will be incorporated into a size- dependent forestry model.

Literature:

Textbooks:

N.Hritonenko and Yu.Yatsenko, *Mathematical Modeling in Economics, Ecology and the Environment. Second Edition*, Springer, 2013

R.Boucekkine, N.Hritonenko, and Yu.Yatsenko (eds.), *Optimal Control of Age-structured Populations in Economy, Demography, and the Environment*, Routledge, 2010.

Additional Literature (Papers):

N.Hritonenko, N. Kato, and Yu.Yatsenko, Optimal control of investments in old and new capital under improving technology, *Journal of Optimization Theory and Applications*, 172 (2017), 247-266

Th. Bréchet, N.Hritonenko, Yu.Yatsenko, Adaptation and mitigation in long-term climate policy, *Environmental and Resource Economics*, 55(2013), 217-243

R. Boucekkine, N.Hritonenko, and Yu.Yatsenko, Optimal investment in heterogeneous capital and technology under restricted natural resource, *Journal of Optimization Theory and Applications* 163 (2014), 310-331.

R.Goetz, N.Hritonenko, R.Mur, A. Xabadia, and Yu.Yatsenko, Forest carbon sequestration as a policy option under climate change, *EM: Magazine for Environmental Managers*, *Air & Waste Management Association,* May 2013, 34–37.

R.Goetz, N.Hritonenko, A. Xabadia, and Yu.Yatsenko, Forest management for timber and carbon sequestration in the presence of climate change: The case of Pinus Sylvestris, *Ecological Economics*, 88(2013), 86-96.

B.Jovanovic and Yu.Yatsenko, Investment in vintage capital, *Journal of Economic Theory*, 147(2012), 551-569.

N.Hritonenko and Yu.Yatsenko, Energy substitutability and modernization of energy-consuming technologies, *Energy Economics,* 34(2012), 1548-1556.

N.Hritonenko, Yu.Yatsenko, R.Goetz, and A. Xabadia, Optimal harvesting in forestry: steady-state analysis and climate change impact, *Journal of Biological Dynamics*, 7(2012), 41-58.

N.Hritonenko and Yu.Yatsenko, Bang-bang, impulse, and sustainable harvesting in age-structured populations, *Journal of Biological Systems,* 20(2012), 133-153.

N.Hritonenko and Yu.Yatsenko, Technological modernization under resource scarcity, *Optimal Control Applications and Methods,* 33(2012), 249-262.

N.Hritonenko and Yu.Yatsenko, Age-structured PDEs in economics, ecology, and demography: Optimal control and sustainability, *Mathematical Population Studies*, 17:4(2010), 191-214.

R.U. Goetz, N.Hritonenko, R.Mur, A.Xabadia, Yu.Yatsenko, Forest management and carbon sequestration in size-structured forests: The case of Pinus Sylvestris in Spain, *Forest Science*, 56:3(2010), 242-256.

N.Hritonenko and Yu.Yatsenko, Maintenance of age-structured populations: optimal control, state constraints, and bang-bang regime, *Journal of Biological Systems*, 17(2009), 793-816.

Mathematical Foundations of Age- and Size-structured Modeling and Control

- **Alexey Davydov & Anton Belyakov**

Mathematical modeling of natural processes is one of the least expensive ways to find out what consequences may result from human activities, for example, the use of renewable resources. The practical need for increasing the efficiency of such processes from an environmental or economic point of view leads to various optimization problems, to the search of control actions, which ensure maximum revenue from the exploitation of renewable resources in the long run.

The course provides an introduction to mathematical models in which the dynamic is described by first order partial differential equation. Beginning with simple concepts, the course provides basic knowledge in the theory of such models, which are quite sufficient for further independent studies.

We also consider essentially nonlinear models of exploitation renewable resources with symmetric (asymmetric) competition, when the models are integro-differential. We discuss the existence of nontrivial stationary solutions under stationary exploitation intensity and the optimal one among them.

The illustrative numerical results will be also presented.

The course will be mainly devoted to theoretical modeling, and, of course, to understand it some knowledge of mathematics is implied, but no any special one is required. Basic knowledge of calculus and numerical methods must be sufficient to understand the material of the course.

Practical exercises in the computer class with MATLAB toolbox for optimal control subject to age-structured systems will be given during the course. Participants will be to find optimal fishing intensity in time that maximizes net present profit from fishing.

Literature:

Arnold, V. I. (2004) Lectures on Partial Differential Equations. Springer Verlag Berlin Heidelberg and PHASIS Moscow 2004. 162 p. ISBN 3-540-40448-1

Courant, R. and Hilbert, D. (1989) General Theory of Partial Differential Equations of First Order, in Methods of Mathematical Physics: Partial Differential Equations, Volume II, Wiley-VCH Verlag GmbH, Weinheim, Germany. doi: 10.1002/9783527617234.ch2

Anita, S. (2000) Analysis and Control of Age-Dependent Population Dynamics, Mathematical Modelling Series. Springer.

Sethi, S.P. & Thompson, G.L. (1981/2000) Optimal Control Theory: Applications to Management Sciences. Nijhoff, Boston.

Belyakov A.O., Veliov V.M. (2014) Constant versus periodic fishing: Age structured optimal control approach Mathematical modeling of natural phenomena. Vol. 9, No. 4, P. 20–37. DOI: 10.1051/mmnp/20149403

Davydov A.A., Nassar A.F. (2015), On the uniqueness of a positive stationary state in the dynamics of a population with asymmetric competition. *Proc. Steklov Inst. Math.*, 291 (2015), 78–86

Age- and Size-structured Modeling of Vegetation Dynamics across Different Scales

Nikolay Strigul

We will consider age-structured models and their generalizations in forest modeling. We will derive these models using the hierarchical patch-dynamics vegetation concept, and discuss underlying assumptions of this modeling approach. The mathematics includes three major components: 1) the use of individual-based models, as they are among the most suitable and promising tools for simulating complex-adaptive systems and interactions on multiple scales, 2) the development of different scaling methods that approximate individual-based processes, and 3) the investigation of various inverse problems to connect models with empirical data. The first component involves mostly computer simulations of what are, in general, analytically intractable stochastic processes. Scaling methods allow models to be reduced to analytically tractable objects–such as different stochastic and deterministic age- and size-structured model–which are both more valuable for experimental scientists and, also, computationally simpler. We will introduce models that are non-linear partial differential or integral equations in case of continuous models, and non-linear recursive and difference equations equivalent to the matrix transition models and Markov Chains in case of discrete models. The mathematical problems that emerge at this stage are quite challenging including analysis of the transient dynamics, stationary states and their stability for these discrete or continuous models. The third component belongs to applied-statistics, and we will briefly overview data-mining methods and challenges involved in parameterization and validation of these models.

Literature:

Strigul N. 2012. Individual-based models and scaling methods for ecological forestry: implications of tree phenotypic plasticity. Sustainable Forest Management, Diez, J.J. (Ed.), InTech, Croatia, 359-384. http://dx.doi.org/10.5772/29590

Strigul N., Pristinski D., Purves D., Dushoff J. & S.W. Pacala. 2008. Scaling from trees to forests: tractable macroscopic equations for forest dynamics. Ecological monographs, 78 (4): 523-545.

Strigul N., Florescu I., Welden A.R. & Michalczewski F. 2012. Modeling of forest stand dynamics using Markov chains. Environmental modelling and software, 31:64-75

Lienard J., & Strigul N. 2016. Modeling of hardwood forest in Quebec under dynamic disturbance regimes: a time-inhomogeneous Markov chain approach. Journal of Ecology, 104(3):806-816

Lienard J., & Strigul N. 2016. An individual-based forest model links canopy dynamics and shade tolerances along a soil moisture gradient. Royal Society Open Science 3 (2), 150589 http://dx.doi.org/10.1098/rsos.150589

Lienard J., Gravel, D. & Strigul N. 2015. Data-intensive multidimensional modeling of forest dynamics. Environmental modelling and software, 67:138-148

Lienard J., Harrison J., & Strigul N. 2016. U.S. Forest Response to Projected Climate-Related Stress: a Tolerance Perspective. Global Change Biology, 22 (8): 2875-2886

Lienard J., Florescu I., & Strigul N. 2015. An appraisal of the classic forest succession paradigm with the shade tolerance index. PLOS One, 10(2): e0117138. http://dx.doi.org/10.1371/journal.pone.0117138

Forest Carbon Sequestration in Continuous Cover Forests

- **Edwin van der Werf**

The world's tropical forests contain enormous stocks of carbon, biodiversity and other ecosystem services. Large parts of these forests are being managed. Forest managers, however, may be primarily interested in timber production. At the same time, policy makers affect the behavior of tropical forest managers, for example through diameter cutting limits or through financial incentives.

We study the management of multi-age multi-species forests in the context of forest carbon sequestration. The literature typically distinguishes matrix growth models, often based on repetitive cycles, and models that allow for continuous harvesting.

Literature:

Assmuth, A., & Tahvonen O. 2018. Optimal carbon storage in even- and uneven-aged forestry. Forest Policy and Economics 87, 93-100

Boscolo, M. & Vincent J.R. 2003. Nonconvexities in the production of timber, biodiversity, and carbon sequestration. Journal of Environmental Economics and Management 46, 251-268

Indrajaya, Y., van der Werf E., Weikard H.-P., Mohren F., & van Ierland E.C. 2016. The potential of REDD+ for carbon sequestration in tropical forests: Supply curves for carbon storage for Kalimantan, Indonesia. Forest Policy and Economics 71, 1-10

Liang, J. & Picard N. 2013. Matrix model of forest dynamics: an overview and outlook. Forest Science 59(3), 359-378

Tahvonen, O. 2015. Economics of naturally regenerating, heterogeneous forests. Journal of the Association of Environmental and Resource Economists 2 (2), 309-337

Research Talks

Liudmila Artemieva: A extra-gradient method for searching for an optimal control in harvesting problems

Anton Belyakov: Optimal cyclic exploitation of renewable resources

Alexey Davydov: Existence of stationary solutions in population dynamic and their optimization

Michael Freiberger: Optimal control in a two stage socio-hydrologicalmodel with random switching time

Natali Hritonenko: Selected mathematical models of environmental processes

Elena Rovenskaya: Balancing ecology and economy in forestry: A theoretical investigation

Nikolay Strigul: Scaling of forest dynamics and self-organization using discrete and continuous models based on conservation laws and age-structured patch mosaic models

Nadezhda Vasilieva: Accounting for non-linear effects in a model of Atlantic cod population and detection of regime shifts

Edwin van der Werf: Logging damage and injured tree mortality in tropical forest management

Scientific Coordinator:

Elena Rovenskaya, Contact: rovenska@iiasa.ac.at

Director, Advanced Systems Analysis (ASA) Program, International Institute for Applied Systems Analysis, Austria

Research Scholar, Optimal Control Department, Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University, Russia